tions," AIAA Education Series, AIAA, Washington, DC, 1989.

[3]Heffley, R. K., Jewell, W. F., Hoh R. H., and Moorhouse, D. J., "Atmospheric Disturbance Models and Requirements for the Flying Qualities Military Standard and Handbook," AIAA Paper 81-0302, Jan. 1981.

[4]Anon., "Flight Dynamics—Concepts, Quantities and Symbols—Part 9: Models of Atmospheric Motions Along the Trajectory of the Aircraft," Draft International Standard ISO/DIS 1151-9, Dec. 1991.

[5]Monteodorisio, T. J., Brown, R. C., Blake, W. B., Flomenhoft, H. I., Jenkins, J. E., Leggett, D. B., Vukelich, S. R., and Zedd, M. F., "Atmospheric and Space Flight Vehicle Coordinate Systems," ANSI/AIAA Recommended Practice, R-004-1992, AIAA, Washington, DC, Feb. 1992.

[6]Anon., "Flight Dynamics—Concepts, Quantities and Symbols—Aircraft Motion Relative to the Air," International Standard ISO 1151-1, Pt. 1 of ISO 1151 Pts. 1–8, International Organization for Standardization, Secretary, Technical Committee 20, Aircraft Industries Assoc., Washington, DC, 1972–1988.

[7]Wagner, H., "Ueber die Entstehung des dynamischen Auftriebes von Tragflügeln," *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 5, Book 1, 1925, pp. 17–35.

[8]Jones, R. T., "The Unsteady Lift of Wings of Finite Aspect Ratio," NACA TR 681, 1940.

[9]Küssner, H. G., "Stresses Produced in Airplane Wings by Gusts," NACA TM 654, Jan. 1932.

[10]Rhode, R. V., and Lundquist, E. E., "Preliminary Study of Applied Load Factors in Bumpy Air," NACA TN 374, April 1931.

[11]Rhode, R. V., "Gust Loads on Airplanes," *Journal of the Society of Automotive Engineers*, Vol. 40, No. 3, 1937, pp. 81–88.

[12]Walker, W. G., and Hadlock, I. K., "An Analysis of the Airspeeds and Normal Accelerations of Boeing B-247 and B-247D Airplanes in Commercial Transport Operations," NACA TN 1783, Dec. 1948.

[13]Donely, P., "Summary of Information Relating to Gust Loads on Airplanes," NACA TR 997, 1950.

[14]Anon., "Airplane Airworthiness," *Civil Aeronautics Manual 04*, Civil Aeronautics Authority, U.S. Dept. of Commerce, Sec. 04.2121, July 1944, p. 25.

[15]Pratt, K. G., and Walker, W. G., "A Revised Gust-Load Formula and a Re-Evaluation of V-G Data Taken on Civil Transport Airplanes from 1933 to 1950," NACA TR 1206, 1954.

[16]Houbolt, J. C., Steiner, R., and Pratt, K. G., "Dynamic Response of Airplanes to Atmospheric Turbulence Including Flight Data on Input and Response," NASA TR R-199, June 1964.

[17]Noback, R., "Comparison of Discrete and Continuous Gust Methods for the Determination of Airplane Design Loads," *Journal of Aircraft*, Vol. 23, No. 3, 1986, pp. 226–231.

Effect of Ground and Ceiling Planes on Shape of Energized Wakes

Vernon J. Rossow*
NASA Ames Research Center,
Moffett Field, California 94035

Nomenclature

b = span of actuator
ds = increment along wake edge
h = height
$\dot{m}$ = mass flow through wake
U_∞, V_∞ = freestream velocity components
u, v = velocity components in x, y directions
x = distance in horizontal direction
y = distance in vertical direction
γ = circulation

Subscripts

c = ceiling
g = ground or floor plane
t = tunnel

Introduction

DURING the planning and design stages of the 80- by 120-ft Wind Tunnel at NASA Ames Research Center, a variety of circuit and test section configurations were considered for the new structure. It was desired that a test section arrangement could be found that would permit the testing of helicopters and augmentors from zero forward velocity, or hover, up to full forward speed. If this were possible, one experimental setup could be used to test the device over its full speed range, thereby saving time and expense to obtain the needed data. In order to accomplish such a goal, the necessary wind-tunnel wall corrections must be available, and the configuration of the test section must be such that excessive interference does not occur. With the technology currently available, interference from the walls of the wind tunnel when powered-lift models are being tested is a difficult but manageable procedure when the freestream velocity is not too small.[1–12] However, as the test section velocity approaches zero, the energized wakes of rotors and augmentors may begin to recirculate so that a portion of the energized wake re-enters the rotor or augmentor. A method of correcting for interference of this kind is not currently available. In order to test at hover and low wind-tunnel speeds, it is therefore necessary that the sidewalls of the wind tunnel have openings that allow the energized wakes to exit the facility without recirculating (Fig. 1). It is then only necessary to determine the factors, or procedure, needed to correct for the presence of the floor (ground) and ceiling planes of the wind tunnel (Fig. 2). In an effort to obtain an estimate of the magnitude and variation of such a correction on the performance of rotors, experiments[13,14] were conducted with a rotor of 12.75 in. diam between large plywood surfaces that represent the ground and ceiling planes of the wind tunnel. Although the configurations tested only covered one blade angle and one rotor diameter, the combination of ground- and ceiling-plane distances were extensive. In addition to the accumulation of data on the variation of thrust with proximity of ground and ceiling planes, it was found that the thrust of the

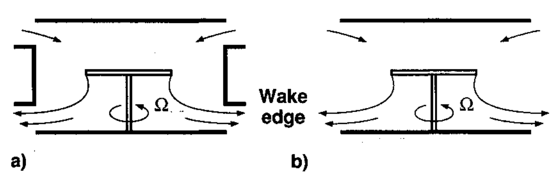

Fig. 1 Wind-tunnel cross sections that illustrate how energized wakes of a rotor can exit the test section without recirculating. Side walls a) partially and b) completely open.

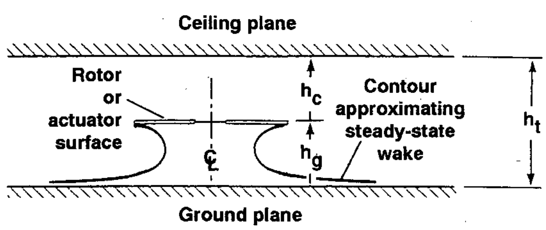

Fig. 2 Cross section of energized wake in the presence of ground and ceiling planes when side walls are not present.

Presented as Paper 93-3410 at the AIAA 11th Applied Aerodynamics Conference, Monterey, CA, Aug. 9–11, 1993; received Aug. 31, 1993; revision received Dec. 31, 1993; accepted for publication Dec. 31, 1993.

*Senior Scientist, Fixed Wing Aerodynamics Branch, Full-Scale Aerodynamics Research Division, M/S N247-2. Associate Fellow AIAA.

rotor changes nearly linearly with the logarithm of distance to the enclosing planes. A theory developed in order to compute wall corrections for the rotor predicted approximately the same characteristic.[13] The analysis was based on the flowfield models introduced by Knight and Hefner,[15,16] and used Bartky's method[17,18] to numerically evaluate the resulting integrals.

Since the previous analysis[13] did not treat the effect of confining surfaces on the shape of the edges of energized wakes, a theoretical study was carried out on that particular aspect of wake structure. In order to limit the scope of the study, attention was restricted to the shape of the edges of energized wakes up near the actuator surface or the beginning of the wake so that the influence of ground and ceiling planes on wake shape could be identified. A representation of the downstream portions of the wakes had to be included, but a complete simulation of all aspects of the dynamics of the wake was not included. That is, it was assumed that the shape of the wake edges up near the actuator surface could be approximated by steady-state, two-dimensional, incompressible, inviscid theory. All unsteady effects and flowfield instabilities are suppressed in the analysis and numerical work. The flowfield model was further idealized so that the energy addition to the wake was uniform over the actuator surface so that vorticity is shed only at the wake edges as shown in Fig. 2, and not throughout.[13] In the theoretical study, both conformal mapping and numerical simulation of the energized wakes were carried out. In this Note, only the highlights of the results are presented and the reader is referred to the preprint version of the paper for more information on the analyses.[19]

Analysis by Simulation of Wake Edges with Singularities

The numerical calculations of the structure of energized, two-dimensional wakes were accomplished by simulating the edges of the energized fluid with either lines of point vortices (all of the same strength), or with segments of vortex sheets connected to form a continuous sheet. Since the wake was assumed to have the same amount of energy added to it everywhere across the actuator surface, vortex sheets are shed only at the edges or boundaries of the wake. The vortex elements are initially placed on lines that roughly approximate the expected wake shape (Fig. 3a). An iterative process is then used to find the steady-state locations of the vortices (Fig. 3b). The locations of the vortices downstream of the ones located at the entry of the wake are determined by the velocity and direction of the streamline through the entry or beginning point of the vortex sheet. An updated position of the vortices is determined by finding the new streamline path through the entry point based on the velocity field of the previous positions of the vortices. In order to suppress instabilities, the vortices are moved from their old positions to the new predicted or updated ones by an amount that ranges from 0.1 to 0.5 of the change in location, depending on the restraint found to be necessary for stability. When boundaries are placed in the flowfield or the spacing of the vortices are decreased, the computation becomes more unstable. A closer spacing (and a corresponding decrease in vortex strength) yields about the same shape for the wake edges, but the solutions become more unstable and the time required for each solution becomes longer. The final locations of the point-vortex arrangement for the finite wake length chosen in Fig. 3 is the same at both ends because the flow circulates. If the length of the vortex wake is much larger than the size of the actuator disk (e.g., 20 to 100 times), the results appear more like a jet of infinite length (Fig. 4).

When the calculations were first made it was found that the steady-state wake configuration was a vortex arrangement, wherein the spacing was approximately constant along the entire wake, not only far from, but also up to, the actuator surface. That is, not only is vorticity produced at a constant rate, but it moves at a constant velocity so that the strength of the vortex sheet is also constant. This result suggests that each wake edge is a boundary between a layer of fluid that is stationary, or nearly so (the ambient fluid), and a layer of energized fluid just inside the wake that is moving at a constant velocity. Fluid further inside or near the center of the wake may, of course, be moving at some other velocity. If it is assumed that the vorticity (or circulation per unit distance) in the wake edge is exactly constant in strength with distance along the wake, and moves at constant velocity, the edges of energized wakes can be treated as free streamlines that move at constant velocity and pressure. With such an approximation, the analysis can be carried out by use of hodograph and mapping methods.[20,21] Such an assumption must have been made by Lighthill[4] when he analyzed the flow of an energized wake as it impinged onto a ground plane.

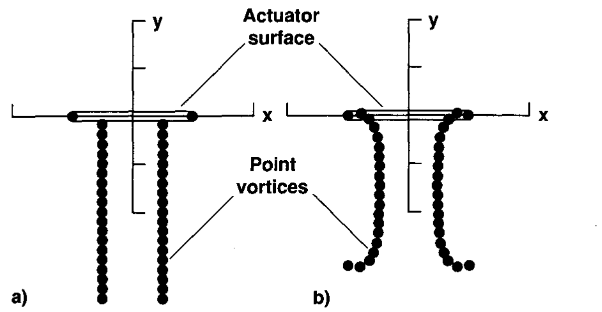

Fig. 3 Energized wake in free space as represented by point vortices: a) initial arrangement of vortices and b) final iterated vortex configuration.

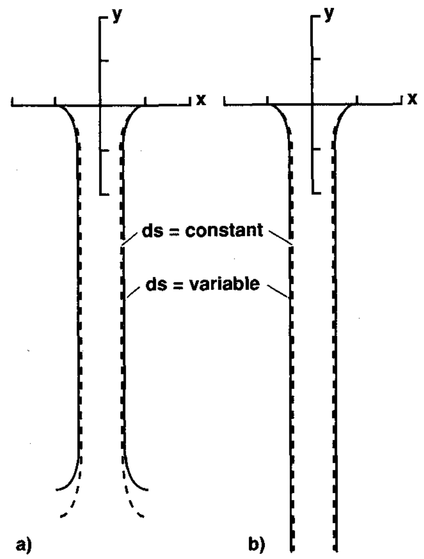

Fig. 4 Comparison of point-vortex representation of energized wake in free space when spacing is held constant and when allowed to be variable. Wake of a) finite and b) infinite length.

As a result of the observation made in the previous paragraph, two methods were used to determine the spacing between vortices in the various sheets that represent wake edges. In the first and most stable method, the vortices are assumed to be spaced at equal distances ds along the streamline as an approximation to the free-streamline approximation (dashed lines in Fig. 4). In this way, only the direction from one point to the next needs to be determined by the computations. In the second method, both the direction and the spacing are allowed to change between vortices. Since a direct approach that adjusts both parameters at once causes the iterations to be less stable, the finalized vortex arrangement found for constant spacing is used for the starting configuration for the variable spacing solution (solid lines in Fig. 4). The fully relaxed or iterated solutions found by the constant and by the variable spacing conditions are illustrated in Fig. 4 for a finite and for a much longer wake. Not much, if any, difference is apparent in the wake shape near the actuator disk or entry region. The fact that the fixed-spacing and the variable-spacing solutions predict a different wake width and length indicates that the velocity and mass flow through the vortex duct are different, even though the strength of the input vorticity at the actuator surface is the same.

Analysis by Use of Free-Streamline Method

As mentioned in the previous section of this Note, numerical analysis of vortex sheets indicates that the spacing of point vortices along the entire sheet is nearly constant. This suggests that the free-streamline method might be a valid approximation to the edges of energized wakes in some circumstances. The advantage of free-streamline solutions is that they have an analytical basis. It is also their main shortcoming because the necessary mapping functions are often difficult or impossible to find and the integrations necessary for a solution may need to be carried out numerically. In brief, the free-streamline method[20,21] assumes that the vortex sheet that represents an edge of the wake is of constant strength along its length. Since the vortex sheet lies along a streamline, there is a velocity, but not a pressure difference across it. Therefore, in the free-streamline approximation, the velocity just outside the sheet is everywhere zero and the velocity along the inside surface of the sheet is everywhere constant. Far downstream from the actuator disk, the velocity across the entire wake is constant.

Of the free-streamline solutions presented in the preprint,[19] only the final result for the solution for the wake in free space is included here (Fig. 5). The boundaries of the flowfield in the hodograph plane are formed by use of the functions[21] that map the real axis into a curvilinear polygon composed of circular arcs. The resulting integrals are then evaluated numerically[17,18] to determine the equipotential and streamlines in the flowfield of an energized wake in free space. The flowfield or physical plane presented in Fig. 5 illustrates the convergence of the wake as it descends. The flow at the entry region where the vortex sheet begins first moves inboard before beginning any downward velocity. The characteristic of wakes to first move inboard is regularly observed in helicopter wakes.[9-12] The amount of convergence of the wake (i.e., 50%) is in agreement with one-dimensional theory.[22] The streamlines show how fluid is drawn into the wake and driven downward. Not quite so obvious is the fact that the velocity just outside of the wake region and close to the vortex sheets is vanishingly small, as assumed in the free-streamline method.

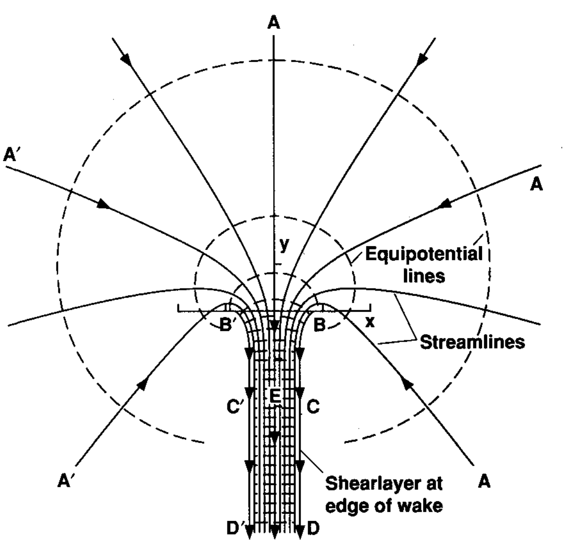

Fig. 5 Physical plane for energized wake in free space.

Discussion of Results

Comparison of Free-Wake Solutions by Different Methods

The comparison in Fig. 6 is made to determine the degree of agreement of the different methods that are used in this Note to predict wake shape for an energized wake in free space with no nearby boundaries or freestream velocity. The wake shape calculated by use of the free-streamline and mapping method is shown as a solid line. The wake shapes predicted by the point-vortex and the vortex-sheet segment methods for an energized wake in free space are shown as a dotted line and as a dashed line, respectively. In these two solutions, the spacing of the point vortices and the lengths of the segments were allowed to vary with distance along the vortex sheets because a variable spacing is the most representative of the wake shape. It is noted in Fig. 6 that all three results predict that the wake edges first move inboard before moving downward. As mentioned previously, this characteristic of wakes is regularly observed in helicopter wakes.[9-12] The agreement in the shape predicted by the three methods (Fig. 6) is quite good everywhere except near the beginning of the sheets where the differences are small. Of the two solutions obtained with singularities, the point vortex is in better agreement with the free-streamline solution than the sheet-segment method. A possible reason for the improved agreement is that a point vortex provides concentrated circulation at the beginning of the vortex sheets which does a better job of supplying the needed flow-turning power than does a segment of vortex sheet. Since the free-streamline solution and the singularity solutions with ds = variable are in quite good agreement with the free-space solution obtained by mapping, a constant velocity assumption for the edges of energized wakes appears to be an approximation that serves to improve the intuitive understanding of energized wakes.

Wake-Shape Changes with Ground and Ceiling Height

Based on the comparisons made in Fig. 6 and in Ref. 19, it is concluded that the point-vortex and vortex-sheet-segment methods yield results acceptably close to the solutions obtained by mapping methods. Since it is much easier to use either of the two singularity methods than the mapping methods, results obtained with the point-vortex method, ds = variable, are chosen for presentation. The presence of a nearby ground or ceiling plane is simulated in the computations by use of an image system of vortices so that the ground plane becomes a streamline. The routine used previously to compute the steady-state vortex positions for a wake in free space is again used here. It was found that greater care is required when an image system of vortices is included if instabilities are to be prevented from occurring.

Consider first the effect of the proximity of the ground plane on wake shape (Fig. 7). It is noted that the thickness of the

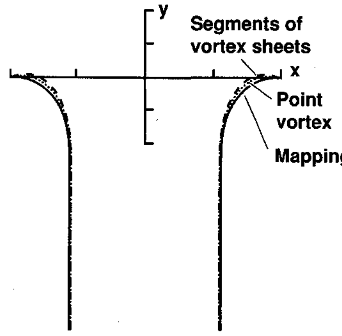

Fig. 6 Comparison of shape of energized wake in free space as calculated by three solution methods.

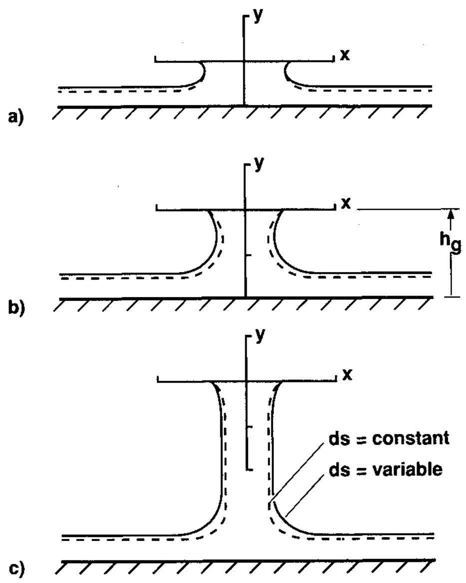

Fig. 7 Typical wake shapes for several locations of ground plane relative to actuator surface as predicted by point-vortex method. h_g/b = a) −0.5, b) −1.0, and c) −2.0.

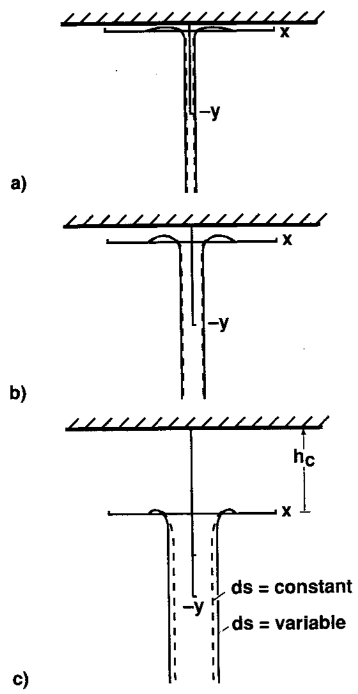

Fig. 8 Typical wake shapes for several locations of ceiling plane relative to actuator surface as predicted by point-vortex method. h_c/b = a) −0.1, b) −0.2, and c) −1.0.

energized wake increases as the ground plane approaches the actuating surface. In contrast, when a ceiling plane approaches the actuator surface, the wake constricts more than when in free space (Fig. 8). As a consequence, the presence of a nearby ceiling plane causes the shear layer to remain in the plane of the actuator much longer than when the wake is in free space. These results suggest that if the wake is generated by a rotor, the noise produced by blade-vortex interactions will probably be aggravated by the presence of a nearby ceiling. Conversely, the presence of a nearby ground plane will tend to alleviate the noise generated. A compromise between ceiling and ground plane distances at which the wake contraction is the same as in free space can probably be found.

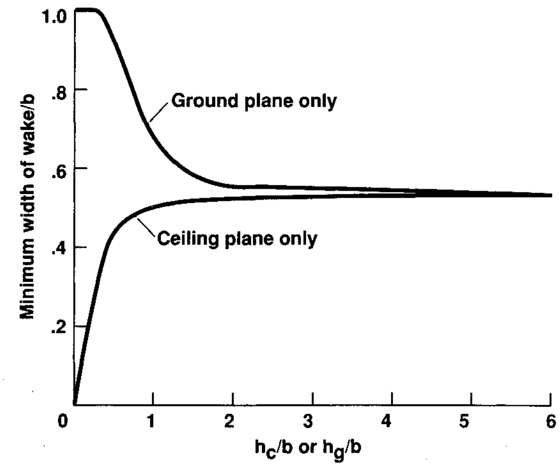

Fig. 9 Minimum width of energized wake as function of the distance of the actuator surface from the ceiling or ground plane.

Other wake characteristics will, however, probably not be the same as the free-space solution.

One way to characterize the effect of ground and ceiling planes on wake shape is to compare the size of the minimum width of the wake as the height of each surface changes (Fig. 9). The results show that a ground plane enlarges the wake width as the actuator disk becomes nearer until the minimum width occurs at the actuator itself. Since such a situation occurs before the ground height h_g vanishes, the minimum wake width is constant for a range of ground heights just above zero. As the actuator surface approaches the ground plane, the mass flow through the wake decreases so that the layers of fluid moving parallel to the ground plane become thinner. As noted previously, the ceiling plane causes the wake to constrict more and more as the actuator approaches the ceiling plane. It is somewhat surprising that, for the idealized, two-dimensional wakes analyzed here, the influence of floor and ceiling planes persists to the large spacings indicated in Fig. 9. The two curves come together with a wake width of about $0.53b$ when the ground and ceiling planes are both about 5 spans from the actuator surface. The proper combination of ground and ceiling heights for minimum influence on rotor thrust was determined in Ref. 13.

Concluding Remarks

Results are presented here that provide insight into the structure of energized wakes in general and in the presence of ground or ceiling planes when powered-lift devices are tested at zero and low wind-tunnel velocities. It was first found that the vortex sheets (that separate the energized wake from the ambient fluid) are approximately constant in strength and move at constant velocity. This characteristic permits the use of free-streamline theory and mapping methods to obtain a solution for an energized wake in free space. It is shown that such a solution is in good agreement with those obtained with point-vortex and vortex-sheet-segment approximations for the wake edges. All three methods indicate that vortex wakes first move inboard, and sometimes also upward, before they turn and go downward, so that interference between the wake and the energizing surface is more likely.

Several solutions were then presented to indicate how the presence of a nearby ground plane causes the energized wake to constrict less than when in free space. In contrast, another set of solutions indicate that the presence of a ceiling plane enhances wake constriction so that the wake width is even smaller than when in free space. Hence, a ground plane tends to reduce the chance for interaction between the wake and the energizing elements, while a nearby ceiling plane tends to increase the likelihood for interference.

References

[1]Heyson, H. H., "Linearized Theory of Wind-Tunnel Jet-Boundary Corrections and Ground Effect for VTOL-STOL Aircraft," NASA TR R-124, 1962.

[2]Betz, A., "The Ground Effect on Lifting Propellers," NACA TM-836, 1937; also *Zeitschrift fuer angewandte Mathematik und Mechanik*, Bd. 17, Nr. 2, April 1937, pp. 68–72.

[3]Greenberg, M. D., and Kaskel, A. L., "Inviscid Flow Field Induced by a Rotor in Ground Effect," NASA CR-1027, May 1968.

[4]Lighthill, J., "A Simple Fluid-Flow Model of Ground Effect on Hovering," *Journal of Fluid Mechanics*, Vol. 93, Pt. 4, 1979, pp. 781–797.

[5]Corsiglia, V. R., Farbridge, J., Dudley, M., and Smith, B. E., "Large-Scale Wind Tunnel Tests of an Ejector-Lift STOVL Aircraft Model," AIAA Paper 89-2905, July 1989.

[6]Dudley, M., Smith, B. E., Corsiglia, V. R., and Ashby, D. L., "Large-Scale Wind Tunnel Studies of an Ejector-Lift STOVL Aircraft Model," 46th Annual Forum of the American Helicopter Society, Paper A-90-46-127-1000, May 1990.

[7]Mueller, R. H. G., "Special Vortices at a Helicopter Rotor Blade," *Journal of the American Helicopter Society*, Vol. 35, No. 4, 1990, pp. 16–22.

[8]Smith, B. E., Garland, D. B., and Poppen, W. A., "Aerodynamic Performance of a Full-Scale Lifting Ejector System in a STOVL Fighter Aircraft," AIAA Paper 92-3094, July 1992.

[9]Gray, R. B., "Vortex Modeling for Rotor Aerodynamics—The 1991 Alexander A. Nikolsky Lecture," *Journal of the American Helicopter Society*, Vol. 37, No. 1, 1992, pp. 3–14.

[10]Bagai, A., and Leishman, J. G., "Improved Wide-Field Shadowgraph Set-Up for Rotor Wake Visualization," *Journal of the American Helicopter Society*, Vol. 37, No. 3, 1992, pp. 86–92.

[11]Bagai, A., and Leishman, J. G., "Experimental Study of Rotor Wake/Body Interactions in Hover," *Journal of the American Helicopter Society*, Vol. 37, No. 4, 1992, pp. 48–57.

[12]Swanson, A. A., "Application of the Shadowgraph Flow Visualization Technique to a Full-Scale Helicopter Rotor in Hover and Forward Flight," AIAA Paper 93-3411, Aug. 1993.

[13]Rossow, V. J., "Effect of Ground and Ceiling Planes on Thrust of Rotors in Hover," NASA TM 86754, July 1985.

[14]Rossow, V. J., "Thrust Changes Induced by Ground and Ceiling Planes on a Rotor in Hover," *Journal of the American Helicopter Society*, Vol. 30, No. 3, 1985, pp. 53–55.

[15]Knight, M., and Hefner, R. A., "Static Thrust of the Lifting Airscrew," NACA TN-626, 1937.

[16]Knight, M., and Hefner, R. A., "Analysis of Ground Effect on the Lifting Airscrew," NACA TN-835, 1941.

[17]Bartky, W., "Numerical Calculation of a Generalized Complete Elliptic Integral," *Reviews of Modern Physics*, Vol. 10, No. 10, 1938, pp. 264–269.

[18]Rossow, V. J., "On Bartky's Method for Evaluation of Integrals of Elliptic Type with Application to Round-Nosed Wedges," *Journal on Scientific and Statistical Computing*, Vol. 6, No. 2, 1985, pp. 365–375.

[19]Rossow, V. J., "Effect of Ground and Ceiling Planes on Shape of Energized Wakes," AIAA Paper 93-3410, Aug. 1993.

[20]Streeter, V. L., *Fluid Dynamics*, McGraw-Hill, New York, 1948, pp. 156–181.

[21]Betz, A., *Konforme Abbildung*, Springer-Verlag, Berlin, 1948, pp. 251–258.

[22]Glauert, H., *The Elements of Aerofoil and Airscrew Theory*, 2nd ed., Cambridge Univ. Press, Cambridge, England, UK, 1948, pp. 199–207.